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Contributions of below–threshold decays to $\mathcal{MSSM}$ Higgs branching ratios.[1]

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Abstract

We calculate all the experimentally relevant branching ratios of the Higgs bosons of the Minimal Supersymmetric Standard Model, paying particular attention to the contributions from below–threshold decays. We show that in some cases these can significantly change the pattern of branching ratios calculated without taking off–shell effects into account.

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1 Introduction

It is likely that the first experimental evidence for supersymmetry will come from studies of the Higgs sector. In supersymmetric extensions of the Standard Model ($\mathcal{SM}$) the Higgs sector invariably has a rich structure, which in the case of the Minimal Supersymmetric Standard Model ($\mathcal{MSSM}$) is also highly constrained (see for example Ref. [1]). Finding one or more of these Higgs bosons is one of the major goals of present and future high–energy colliders.

Many years of phenomenological studies [2, 3, 4, 5, 6, 7, 8] have produced several important lessons for Higgs searches. The first is that almost all decay channels have large $\mathcal{SM}$ backgrounds. It is therefore important to have very precise calculations for the Higgs production cross sections and branching ratios, so that small signals can be unambiguously identified. A second lesson is that 'below–threshold' decays can be very important. By this we mean Higgs decays via intermediate states in which one or more particles are off–mass–shell. A simple example of this is provided by the $\mathcal{SM}$ Higgs decay $\phi \to Z^0 Z^0 \to 4l^\pm$. In the 'on–shell' approach, one would calculate the branching ratio for this as $BR(\phi \to Z^0 Z^0) \times BR(Z^0 \to l^+ l^-)^2$, with $BR(\phi \to Z^0 Z^0) = 0$ for $M_\phi < 2M_{Z^0}$. This, however, is too naive. The below–threshold 'off–shell' decay $\phi \to Z^0 (Z^0)^* \to 4l^\pm$ is non–negligible, and in fact provides a very important Higgs signature for $O(130 \text{ GeV}) < M_\phi < 2M_{Z^0}$.

Although below–threshold decays have been extensively studied (see below) for the $\mathcal{SM}$, to date there has been no equivalent study of the $\mathcal{MSSM}$. In fact all $\mathcal{MSSM}$ Higgs phenomenological studies have so far used branching ratios based on on–shell calculations (except for the trivial cases of $H \to W^\pm (W^\mp)^*, Z^0 (Z^0)^*$ which are easily obtained from the corresponding $\mathcal{SM}$ results). In this paper we present a complete calculation[2] of all relevant $\mathcal{MSSM}$ Higgs branching ratios, paying particular attention to below–threshold contributions. Of course it is impossible to cover *all* regions of the $\mathcal{MSSM}$ parameter space. Our aim is to identify those situations where below–threshold decays are important, and those where simple on–shell calculations are sufficient. We illustrate our results by numerical calculations using 'typical' parameter values. Our aim is to provide the necessary theoretical tools for precision $\mathcal{MSSM}$ Higgs phenomenology at future colliders.

The paper is organized as follows. In the next section we give a comprehensive review

[2] For simplicity we ignore all direct decays involving supersymmetric particles

of $\mathcal{SM}$ and $\mathcal{MSSM}$ branching ratios. In Section 3 we present numerical results for some typical $\mathcal{MSSM}$ scenarios, highlighting the differences between off–shell and on–shell calculations. In Section 4 we present our conclusions.

2 $\mathcal{SM}$ and $\mathcal{MSSM}$ Higgs Branching Ratios

The branching ratios of the $\mathcal{MSSM}$ Higgs bosons have been studied in various papers. A good review of the early works on this subject can be found in Ref. [1], where all the most relevant formulae for on–shell decays have been summarized. These include the $\mathcal{SM}$–like two–body decays[3] $\Phi \to f\bar{f}$ (where f represents a generic massive fermion) [9], $H \to W^{\pm}W^{\mp}, Z^0Z^0$ [10] at tree–level, and the one–loop induced decays $\Phi \to Z^0\gamma, \gamma\gamma$ [11] and gg [12], generalized to the $\mathcal{MSSM}$ [13, 14]; and the $\mathcal{MSSM}$–specific decays $H^{\pm} \to f\bar{f}'$, $A \to Z^0h$, $H^{\pm} \to W^{\pm}h$, $H \to hh, AA$, $\Phi \to \tilde{\chi}_i^{\pm}\tilde{\chi}_j^{\mp}$ (chargino decays) and $\Phi \to \tilde{\chi}_i^0\tilde{\chi}_j^0$ (neutralino decays) [13]. At the time of Ref. [1] only the tree–level Higgs mass relations had been computed, and the on–shell decays $h \to AA$ and $H \to H^{\pm}H^{\mp}, Z^0A$ were kinematically forbidden. We now know that they are allowed at one–loop and the corresponding formulae can be easily obtained, for example from Ref. [13]. Finally, the one–loop induced decays of the charged Higgses $H^{\pm} \to W^{\pm}Z^0, W^{\pm}\gamma$ have also been studied [15].

Recently, higher–order corrections to most of these processes have been computed. For the $\mathcal{SM}$, one can find in the literature the QCD [16] and EW [17, 18, 19, 20] radiative corrections, and their interplay [21, 22] for the $f\bar{f}$–channels. The EW corrections to the $W^{\pm}W^{\mp}, Z^0Z^0$ decay rates [18, 23, 24], and both the QCD [25, 26] and EW [24] corrections to the $Z^0\gamma, \gamma\gamma, gg$ decay rates are also now available. A detailed and updated review of the higher–order corrections computed to date within the $\mathcal{SM}$ can be found in Ref. [27].

As at tree–level, the higher–order $\mathcal{SM}$ corrections can easily be extended to the $\mathcal{MSSM}$ case. For the $\mathcal{MSSM}$–specific processes, both the QCD [28] and the EW [29] corrections to the fermionic decays $H^{\pm} \to f\bar{f}'$ of the charged Higgs bosons, and the EW corrections [24] to the decays involving neutral bosons (such as $H \to hh$ and $A \to Z^0h$) have been computed.

Within the $\mathcal{MSSM}$, a systematic analysis of all the interesting decays of both neutral

[3]Throughout this study we use Φ to denote the set of neutral $\mathcal{MSSM}$ Higgs bosons h, H and A

and charged Higgs bosons, with reference to the search strategies at the LHC proton–proton collider and taking certain of the above results into account, has been presented in Ref. [30], using tree–level formulae for masses and couplings. This analysis has been updated in Ref. [31] to include new results for the one–loop corrections to the latter [32, 33]. Similar studies have been reported in Refs. [34, 35, 36], and extended to the LEP I and LEP II e^+e^- colliders in Ref. [37]. For a NLC e^+e^- collider the corresponding analysis has been performed in Ref. [38].

Apart from the processes $H \to W^{\pm *}W^{\mp *}, Z^{0*}Z^{0*}$ [39], the results on decay widths and branching ratios reported in Refs. [31, 34, 35, 36, 37, 38] have been obtained for the case of above–threshold decays only. It is our aim in this paper to generalize these results to include also below–threshold decays. In particular, we study the two–body decay channels

$$\Phi \to s\bar{s}, c\bar{c}, b\bar{b}, t^*\bar{t}^*, \mu^+\mu^-, \tau^+\tau^-, \tag{1}$$

$$\Phi \to W^{\pm *}W^{\mp *}, Z^{0*}Z^{0*}, Z^{0*}\gamma, \gamma\gamma, gg, \tag{2}$$

$$H \to h^*h^*, \qquad H \to A^*A^*, \qquad A \to Z^{0*}h^*, \tag{3}$$

for the neutral Higgses $\Phi = H$, h and A, and

$$H^+ \to c\bar{s}, t^*\bar{b}, \mu^+\nu_\mu, \tau^+\nu_\tau, \tag{4}$$

$$H^+ \to W^{+*}h^*, \tag{5}$$

for the charged Higgses $H^\pm$'s. We do not consider (i) the tree–level decays

$$H \to Z^{0*}A^*, \qquad H \to H^{+*}H^{-*}, \qquad h \to A^*A^*, \tag{6}$$

since these are only possible in regions of the $\mathcal{MSSM}$ $(M_A, \tan\beta)$ parameter space already excluded by LEP I (for $m_t < 200$ GeV) [40], and (ii) the one–loop induced decays

$$H^+ \to W^{+*}\gamma, \qquad H^+ \to W^{+*}Z^{0*}, \tag{7}$$

since these have very small branching ratios [15].

In calculating the decay rates for the processes (1)–(5) we have adopted the off–shell decay formulae presented in Ref. [41] for $\Phi \to W^*W^*, Z^{0*}Z^{0*}$ and in Refs. [42, 43] for $\Phi \to Z^{0*}\gamma$. For the other cases not involving decays to top quarks we have recalculated the off–shell formulae at tree–level, tracing the corresponding amplitudes squared and integrating

analytically over the appropriate off–shell phase space. We then integrate numerically over all the possible virtualities of the final–state particles for all the processes. This is in contrast to the procedure sometimes used of constraining one of the decay products to be on–shell when the decaying Higgs boson mass exceeds its rest mass. Although this allows the width to be computed analytically, it can give misleading results near threshold. Our procedure is only to use such 'integrated' analytic expressions well above threshold. We therefore proceed in the following way. With A the decaying particle and $B^{(*)}$ and $C^{(*)}$ the decay products, which can be either on–shell (B, C) or off–shell (B^*, C^*), we define as the decay width of the channel $A \to B^{(*)}C^{(*)}$ the function

$$\Gamma(A \to B^{(*)}C^{(*)}) = \int\int_R \frac{\mathrm{d}Q_B^2 M_B \Gamma_B}{\pi[(Q_B^2 - M_B^2)^2 + (\alpha_B \Gamma_B)^2]} \times \frac{\mathrm{d}Q_C^2 M_C \Gamma_C}{\pi[(Q_C^2 - M_C^2)^2 + (\alpha_C \Gamma_C)^2]} \times \Gamma(A \to B^* C^*), \tag{8}$$

where $M_{B(C)}$ and $\Gamma_{B(C)}$ are the mass and the width of the particle $B(C)$ with four–momentum squared $Q^2_{B(C)}$, respectively, and $\Gamma(A \to B^*C^*)$ the off–mass–shell decay width into B and C. The two–dimensional integration region R is defined by

$$Q_B \geq m_b, \quad Q_C \geq m_c, \quad Q_B + Q_C \leq M_A, \tag{9}$$

where $m_{b(c)}$ represents the sum of the rest masses of the decay products of the particle $B(C)$. The change of variables [44]

$$Q_X^2 - M_X^2 = M_X \Gamma_X \tan\theta_X, \quad \Longrightarrow \mathrm{d}Q_X^2 = \frac{(Q_X^2 - M_X^2)^2 + \alpha_X^2 \Gamma_X^2}{M_X \Gamma_X}\,\mathrm{d}\theta_X, \quad X = B, C, \tag{10}$$

then gives an integrand smoothly dependent on the integration variables. For the numerical evaluation we use VEGAS [45].

The expression given in (8) is a good approximation when $M_X \gg m_x$ $(X = B, C)$ so that $\Gamma_X \propto M_X$. We therefore use it for decays to vector–vector, vector–scalar and scalar–scalar pairs. It is *not* however a good approximation for decays involving the top quark, e.g. $\Phi \to t\bar{t}$ or $H^+ \to t\bar{b}$. For such decays we must perform an exact matrix element calculation for the complete process, e.g. $\Phi \to bW^+\bar{b}W^-$ or $H^+ \to bW^+\bar{b}$.

We are not interested here in studying in detail the dependence of the processes (1)–(5) on all possible higher–order corrections to the widths and branching ratios. Instead we wish to find out which below–threshold $\mathcal{MSSM}$ Higgs decays are important. We therefore include only the larger non–SUSY QCD corrections (when known), and not the EW ones.

In any case, our procedure can readily be adapted to include such corrections if necessary. We have included the modifications due to QCD loops in the computations following Ref. [16] for the processes $\Phi \to q\bar{q}$, Ref. [28] for $H^\pm \to q\bar{q}'$, Ref. [25] for $h \to \gamma\gamma, Z^0\gamma$ and, finally, Ref. [26] for $h \to gg$. In particular, since the QCD corrections to the top loops in $Z^0\gamma$, $\gamma\gamma$ and gg decays, as given in Refs. [25, 26], are valid for $M_\Phi < 2m_t$, we have implemented these only for the lightest $\mathcal{MSSM}$ neutral Higgs, i.e. $\Phi = h$. We do not include any threshold effects due to the possible formation of $t\bar{t}$–bound states in the one–loop induced processes, when the Higgs mass M_Φ reaches a value $\approx 2m_t$ [46].

There is a slight subtlety concerning the 'running' of the fermion masses. For the decays involving light quarks $\Phi \to q\bar{q}$ ($q = s, c, b$), the use of the running quark mass $m_q^2(Q^2 = M_\Phi^2)$ takes into account large logarithmic corrections at higher orders in QCD perturbation theory [16]. In principle, one could imagine using the same procedure for $\Phi \to t\bar{t}$ in the limit $M_\Phi \gg m_t$. In practice, however, we are interested in the case of $M_\Phi/m_t \sim \mathcal{O}(1)$. For the top–loop mediated decay $\Phi \to gg$, it is known that the higher–order QCD corrections are minimized if the quark mass is defined at the pole of the propagator, i.e. $m_t^2(Q^2 = m_t^2)$ [47]. For the *pseudoscalar* Higgs, the decay rate for $A \to gg$ has a local maximum at $M_A = 2m_t$. To be consistent, therefore, we must use the *same* top mass ($m_t^2(Q^2 = m_t^2)$) in the decay rate for $\Phi \to t\bar{t}$.

To avoid a proliferation of additional parameters, we neglect small chargino and neutralino contributions in the $\Phi \to Z^0\gamma$, $\gamma\gamma$ decays. To further simplify the discussion, we assume a universal soft supersymmetry–breaking mass [32, 33]

$$m_Q^2 = m_U^2 = m_D^2 = m_{\tilde{q}}^2, \tag{11}$$

and negligible mixing in the *stop* and *sbottom* mass matrices,

$$A_t = A_b = \mu = 0. \tag{12}$$

Moreover, if we also neglect the b–quark mass in the formulae of Refs. [32, 33], the one–loop corrected masses of the $\mathcal{MSSM}$ neutral $\mathcal{CP}$–even Higgs bosons can be expressed in terms of a single parameter ϵ [37], given by

$$\epsilon = \frac{3e^2}{8\pi^2 M_{W^\pm}^2 \sin^2\theta_W}\, m_t^4 \ln\left(1 + \frac{m_{\tilde{q}}^2}{m_t^2}\right). \tag{13}$$

Diagonalization of the mass–squared matrix leads to the expressions

$$M_{h,H}^2 \;=\; \frac{1}{2}[M_A^2 + M_{Z^0}^2 + \epsilon/\sin^2\beta]$$

$$\pm\left\{[(M_A^2 - M_{Z^0}^2)\cos 2\beta + \epsilon/\sin^2\beta]^2 + (M_A^2 + M_{Z^0}^2)^2 \sin^2 2\beta\right\}^{1/2}, \qquad (14)$$

while the mixing angle α in the $\mathcal{CP}$–even sector is defined at one–loop by

$$\tan 2\alpha = \frac{(M_A^2 + M_{Z^0}^2)\sin 2\beta}{(M_A^2 - M_{Z^0}^2)\cos 2\beta + \epsilon/\sin^2\beta}, \qquad -\frac{\pi}{2} < \alpha \leq 0. \qquad (15)$$

For the $\mathcal{MSSM}$ charged Higgs masses we have maintained the tree–level relations

$$M_{H^\pm}^2 = M_A^2 + M_{W^\pm}^2, \qquad (16)$$

since one–loop corrections are quite small in comparison to those for the neutral Higgs bosons [33].

For the above choice of parameters we also use the leading one–loop corrected expressions for the Hhh and HAA trilinear couplings, which enter in the Higgs branching ratios studied here, i.e.

$$\lambda_{Hhh} = \lambda_{Hhh}^0 + \Delta\lambda_{Hhh}, \qquad (17)$$

$$\lambda_{HAA} = \lambda_{HAA}^0 + \Delta\lambda_{HAA}, \qquad (18)$$

with the tree–level relations

$$\lambda_{Hhh}^0 = -\frac{igM_{Z^0}}{2\cos\theta_W}[2\sin(\beta+\alpha)\sin 2\alpha - \cos(\beta+\alpha)\cos 2\alpha], \qquad (19)$$

$$\lambda_{HAA}^0 = \frac{igM_{Z^0}}{2\cos\theta_W}\cos 2\beta\cos(\beta+\alpha), \qquad (20)$$

and the one–loop corrections

$$\Delta\lambda_{Hhh} = -\frac{igM_{Z^0}}{2\cos\theta_W}\frac{3g^2\cos^2\theta_W}{8\pi^2}\frac{\cos^2\alpha\sin\alpha}{\sin^3\beta}\frac{m_t^4}{m_{W^\pm}^4}\left(3\log\frac{m_{\tilde{q}}^2 + m_t^2}{m_t^2} - 2\frac{m_{\tilde{q}}^2}{m_{\tilde{q}}^2 + m_t^2}\right), \qquad (21)$$

$$\Delta\lambda_{HAA} = -\frac{igM_{Z^0}}{2\cos\theta_W}\frac{3g^2\cos^2\theta_W}{8\pi^2}\frac{\sin\alpha\cos^2\beta}{\sin^3\beta}\frac{m_t^4}{m_{W^\pm}^4}\log\frac{m_{\tilde{q}}^2 + m_t^2}{m_t^2}, \qquad (22)$$

as given in Ref. [31].

In the numerical calculations presented in the following section we adopt the following values for the electromagnetic coupling constant and the weak mixing angle, $\alpha_{em} = 1/128$ and $\sin^2\theta_W = 0.23$, respectively. The strong coupling constant α_s, which appears at leading order in the gg decay widths and also enters via the QCD corrections, has been evaluated at two loops, with $\Lambda_{\overline{\rm MS}}^{(4)} = 190$ MeV, and with the number of active flavours N_f

(and the corresponding $\Lambda_{\overline{\rm MS}}^{(N_f)}$, calculated according to the prescription of Ref. [48]) and scale Q^2 chosen appropriately for the decay in question, i.e. adopting the prescriptions of Refs. [16, 25, 26, 28].

For the gauge boson masses and widths we take $M_{Z^0} = 91.1$ GeV, $\Gamma_{Z^0} = 2.5$ GeV, $M_{W^\pm} = M_{Z^0}\cos\theta_W$ and $\Gamma_{W^\pm} = 2.2$ GeV, while for the fermion masses we take $m_\mu = 0.105$ GeV, $m_\tau = 1.78$ GeV, $m_s = 0.3$ GeV, $m_c = 1.4$ GeV, $m_b = 4.25$ GeV and $m_t = 175$ GeV [49], with all widths equal to zero except for Γ_t. We calculate this at tree–level within the $\mathcal{MSSM}$, using the expressions (for the above values of m_t and m_b) [50]

$$\begin{aligned}\frac{\Gamma(t\to bH^+)}{\Gamma(t\to bW^+)} &= \frac{\lambda(M^2_{H^\pm},m_b^2,m_t^2)^{1/2}}{\lambda(M^2_{W^\pm},m_b^2,m_t^2)^{1/2}}\times \\ &\quad \frac{(m_t^2+m_b^2-M^2_{H^\pm})(m_t^2\mathrm{cotan}^2\beta+m_b^2\tan^2\beta)+4m_t^2m_b^2}{M^2_{W^\pm}(m_t^2+m_b^2-2M^2_{W^\pm})+(m_t^2-m_b^2)^2},\end{aligned} \tag{23}$$

and [51]

$$\begin{aligned}\Gamma(t\to bW^+) &= |V_{tb}|^2\frac{G_F m_t}{8\sqrt{2}\pi}\lambda(M^2_{W^\pm},m_b^2,m_t^2)^{1/2}\times \\ &\quad \left\{\left[1-\left(\frac{m_b}{m_t}\right)^2\right]^2+\left[1+\left(\frac{m_b}{m_t}\right)^2\right]\left(\frac{M_{W^\pm}}{m_t}\right)^2-2\left(\frac{M_{W^\pm}}{m_t}\right)^4\right\},\end{aligned} \tag{24}$$

where V_{tb} is the Cabibbo–Kobayashi–Maskawa mixing angle (here set equal to 1), $G_F = \sqrt{2}g^2/8M^2_{W^\pm}$ is the electroweak Fermi constant, with $g = e/\sin\theta_W$ and $-e$ the electron charge, and $\lambda^{1/2}$ is the usual kinematic factor

$$\lambda(M_a,M_b,M_c)^{1/2} = [M_a^2+M_b^2+M_c^2-2M_aM_b-2M_aM_c-2M_bM_c]^{1/2}. \tag{25}$$

The first generation of fermions and all neutrinos are taken to be massless, i.e. $m_u = m_d = m_e = m_{\nu_e} = 0$ and $m_{\nu_\mu} = m_{\nu_\tau} = 0$, with zero decay widths. Finally, the universal supersymmetry–breaking squark mass is in the numerical analysis to be $m_{\tilde q} = 1$ TeV.

3 Numerical results

It is impractical to cover all possible regions of the $\mathcal{MSSM}$ parameter space. We choose a representative set of figures which (a) describes the decay channels and mass ranges which are relevant to experiment, (b) illustrates the importance of below–threshold decays, and (c) allows a comparison with previous studies. Thus Figs. 1 – 4 show the branching ratios

for the h, H, A and $H^{\pm}$ $\mathcal{MSSM}$ Higgs bosons respectively, as a function of the mass of the decaying particle. Following Ref. [31], we choose two representative values, $\tan\beta = 1.5, 30$, which give significant differences in certain channels. In comparing our figures with those of Ref. [31], it should be remembered that we have chosen a top quark mass ($m_t = 175$ GeV) consistent with the recent CDF measurement.

Figure 1 shows the branching ratios of the lightest neutral scalar $\mathcal{MSSM}$ Higgs boson h as a function of M_h. Note that for this value of m_t, the maximum values of M_h are 98(129) GeV for $\tan\beta = 1.5(30)$, achieved in the limit $M_A \to \infty$. As expected, the $b\bar{b}$ and $\tau^+\tau^-$ channels dominate over essentially the complete mass range. The $c\bar{c}$ and $t\bar{t}$ branching ratios are smaller for larger $\tan\beta$, reflecting the dependence of the $hf\bar{f}$ couplings on the angles (α, β).

The new feature of Fig. 1 compared with the corresponding figure (Fig. 10) of Ref. [31] is the appearance of the below–threshold decays $h \to W^+W^-,\ Z^0Z^0$, especially for smaller $\tan\beta$ values. In fact for $\tan\beta = 1.5$, the W^+W^- branching ratio is larger than that for $\gamma\gamma$ over a sizeable portion of the h mass range (when $M_h \gtrsim 88$ GeV). Fortunately, because of their negligible contribution to the total width compared to the dominant modes, the two new channels do not depress the $\gamma\gamma$ branching ratio, which is very important for Higgs searches at the LHC proton–proton collider. This does happen at $\tan\beta = 30$ (and also affects the $b\bar{b}$ and $\tau^+\tau^-$ channels), but only in a very narrow mass window, and so is unlikely to be phenomelogically relevant. Note that the quasi–singular behaviour of the branching ratios near $M_h = M_h^{\rm max}$ is caused by the mapping of a large range of M_A masses into a small range of M_h masses, see Ref. [31] for a fuller discussion.

In Fig. 2 we show the corresponding set of branching ratios for the heavier of the two neutral scalar $\mathcal{MSSM}$ Higgs bosons H. This has a rather complicated structure which is discussed in Ref. [31]. In summary, for large $\tan\beta$ the only important decays are to $b\bar{b}$ and $\tau^+\tau^-$, apart from a very small mass region around $M_H = M_H^{\rm min}$ where other decay channels are important. For smaller $\tan\beta$, either the W^+W^-, hh or $t\bar{t}$ channels dominate, depending on M_H. The inclusion of below–threshold decays has little effect here, serving mainly to generate a smooth transition around the $t\bar{t}$ threshold region.

Figure 3 shows the branching ratios for the pseudoscalar A Higgs boson. Here the difference between large and small $\tan\beta$ is again very marked. For the former, the $b\bar{b}$ and $\tau^+\tau^-$ decays are completely dominant, and below–threshold decay effects are negligible. For the latter, the $t\bar{t}$ decay dominates above threshold and the Z^0h decay channel can

also be important. The point which we wish to stress is that the below–threshold $t\bar{t}$ decay plays an important role. To see this more clearly, Fig. 4 shows the corresponding set of branching ratios when below–threshold decays are forbidden. (This is the analogue of Fig. 12 in Ref. [31].) In this case for $\tan\beta = 1.5$ there are three distinct mass regions: $M_A < (M_Z + M_h)$, $(M_Z + M_h) < M_A < 2m_t$ and $2m_t < M_A$ where the dominant decay modes are $b\bar{b}$, $Z^0 h$ and $t\bar{t}$ respectively. However, the middle mass region is strongly affected by the below–threshold $t\bar{t}$ decay. In particular, for 300 GeV $< M_A <$ 350 GeV the $b\bar{b}$ and $Z^0 h$ decay modes are suppressed far below their 'naive' values shown in Fig. 4. This clearly has important phenomenological implications, especially since the leading $A \to t\bar{t} \to bW^+\bar{b}W^-$ channel is not readily observable.

Another interesting effect is seen in the charged Higgs branching ratios, Fig. 5. At large $\tan\beta$ the only relevant decays are tb and $\tau\nu$. Here the inclusion of the below–threshold decay for the former simply gives a smoother transition between the two; compare Fig. 14(b) of Ref. [31]. At smaller $\tan\beta$ there is a window (assuming the CDF central value for m_t) for the Wh channel to be important. In fact this would be the dominant channel over a sizeable range of M_A in the absence of the below–threshold tb decay. However, we see that when the threshold behaviours are correctly taken into account the tb decay rate is almost always larger than that for the the Wh channel. Again, this has important implications for charged Higgs searches at future colliders.

4 Conclusions

In summary, we have performed a comprehensive study of the phenomenologically relevant decay rates of the $\mathcal{MSSM}$ Higgs bosons, paying particular attention to below–threshold decay channels. We have discovered several cases where these may be important. For $\tan\beta$ not too large, there are non-neglible branching ratios for the decay of the lightest neutral Higgs boson h into off-shell W^+W^- and Z^0Z^0. At the upper end of the M_h mass range these can reach $\mathcal{O}(10^{-3} - 10^{-2})$ and (in the case of W^+W^-) can be larger than the important $\gamma\gamma$ branching ratio.

For the decays of heavier neutral bosons H and A, the main impact is from below–threshold $t\bar{t}$ decays. Again for modest values of $\tan\beta$ these can suppress the branching ratios of phenomenologically important channels such as $H \to hh, W^+W^-, Z^0Z^0, b\bar{b}$ and $A \to Zh, b\bar{b}, \tau^+\tau^-$ below their values calculated without taking the off-shell effects into

account (compare Figs. 3 and 4). In the same way, the below–threshold decay $H^{\pm} \to tb$ of the charged Higgs boson can suppress the branching ratio of the important $H^{\pm} \to W^{\pm}h$ channel.

As the physics studies for future high-energy colliders become more focused, it is important that all the ingredients of 'new physics' search strategies are calculated with as high a precision as possible. We believe that our study of $\mathcal{MSSM}$ Higgs boson decays provides a significant improvement in this respect.

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Figure captions

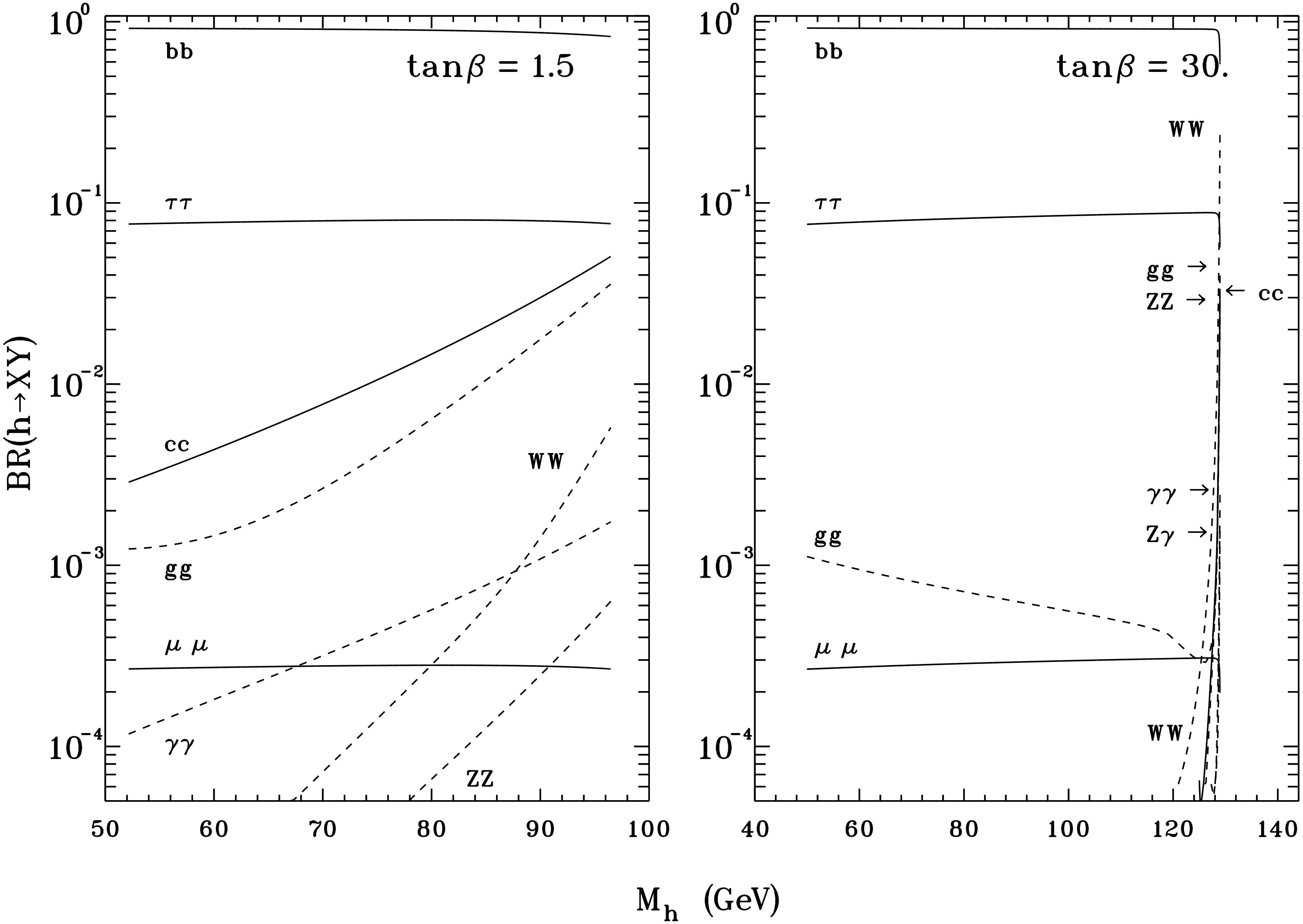
[1] Branching ratios for the lightest neutral scalar $\mathcal{MSSM}$ Higgs boson h as a function of M_h, for $\tan\beta = 1.5$ and 30. Other parameter values are given in the text.

[2] Branching ratios for the heavier of the two neutral scalar $\mathcal{MSSM}$ Higgs bosons H as a function of M_H, for $\tan\beta = 1.5$ and 30. Other parameter values are given in the text.

[3] Branching ratios for the pseudoscalar $\mathcal{MSSM}$ Higgs bosons A as a function of M_A, for $\tan\beta = 1.5$ and 30. Other parameter values are given in the text.

[4] As for Fig. 3, but with the below–threshold decays omitted.

[5] Branching ratios for the charged $\mathcal{MSSM}$ Higgs bosons $H^{\pm}$ as a function of $M_{H^{\pm}}$, for $\tan\beta = 1.5$ and 30. Other parameter values are given in the text.

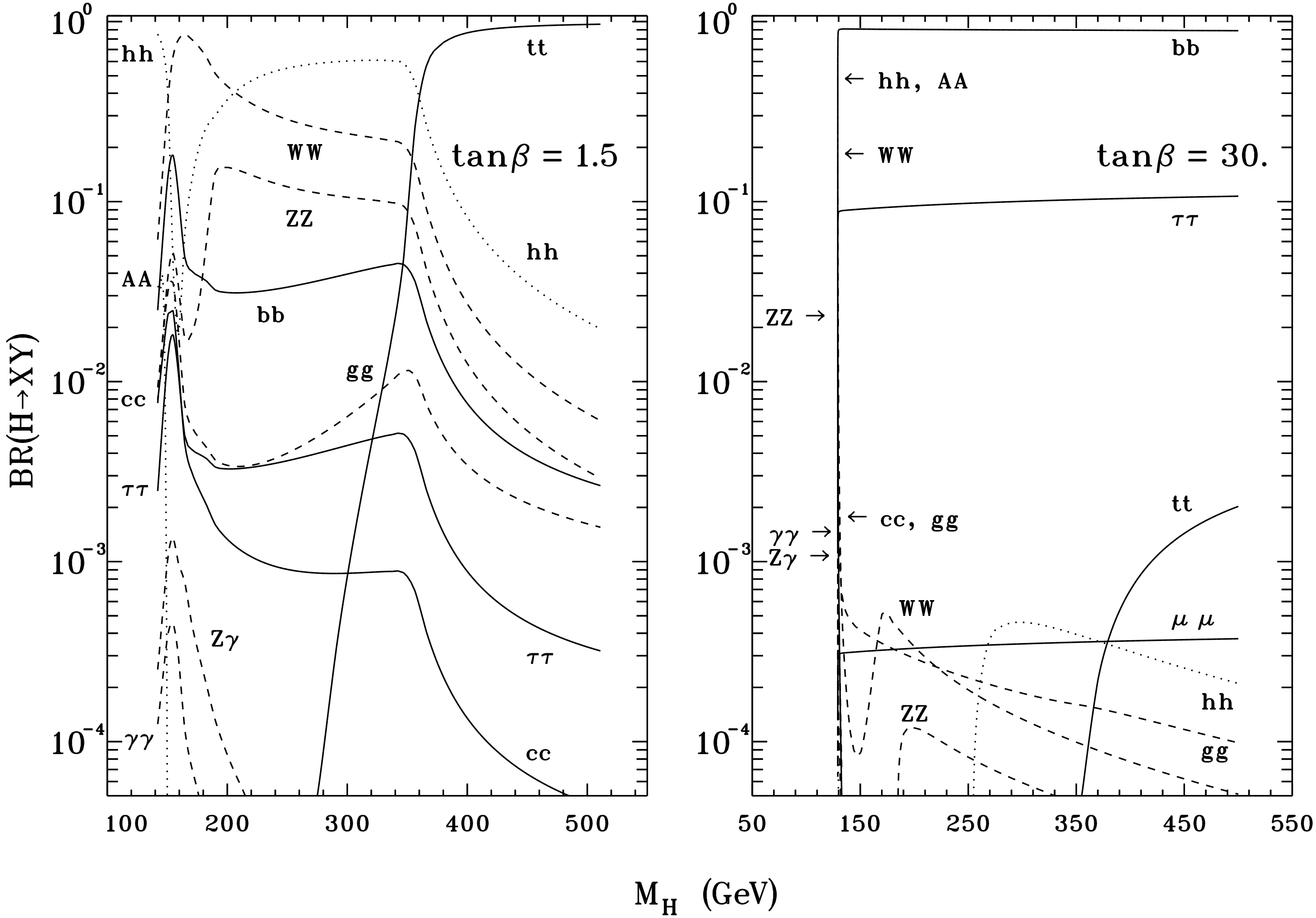


BR(h→XY)
tanβ = 1.5
bb
ττ
cc
gg
μ μ
γγ
WW
ZZ
tanβ = 30.
bb
ττ
WW
gg →
ZZ →
← cc
γγ →
Zγ →
gg
μ μ
WW
M_h (GeV)

Fig. 1

This figure "fig1-2.png" is available in "png" format from:

http://arXiv.org/ps/hep-ph/9412209v1



10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
BR(H→XY)
$\tan\beta = 1.5$
hh
tt
WW
ZZ
hh
AA
bb
gg
cc
$\tau\tau$
$Z\gamma$
$\tau\tau$
$\gamma\gamma$
cc
100
200
300
400
500
$\tan\beta = 30.$
bb
← hh, AA
← WW
$\tau\tau$
ZZ →
← cc, gg
$\gamma\gamma$ →
$Z\gamma$ →
tt
WW
$\mu\ \mu$
hh
ZZ
gg
50
150
250
350
450
550
M_H (GeV)

Fig. 2

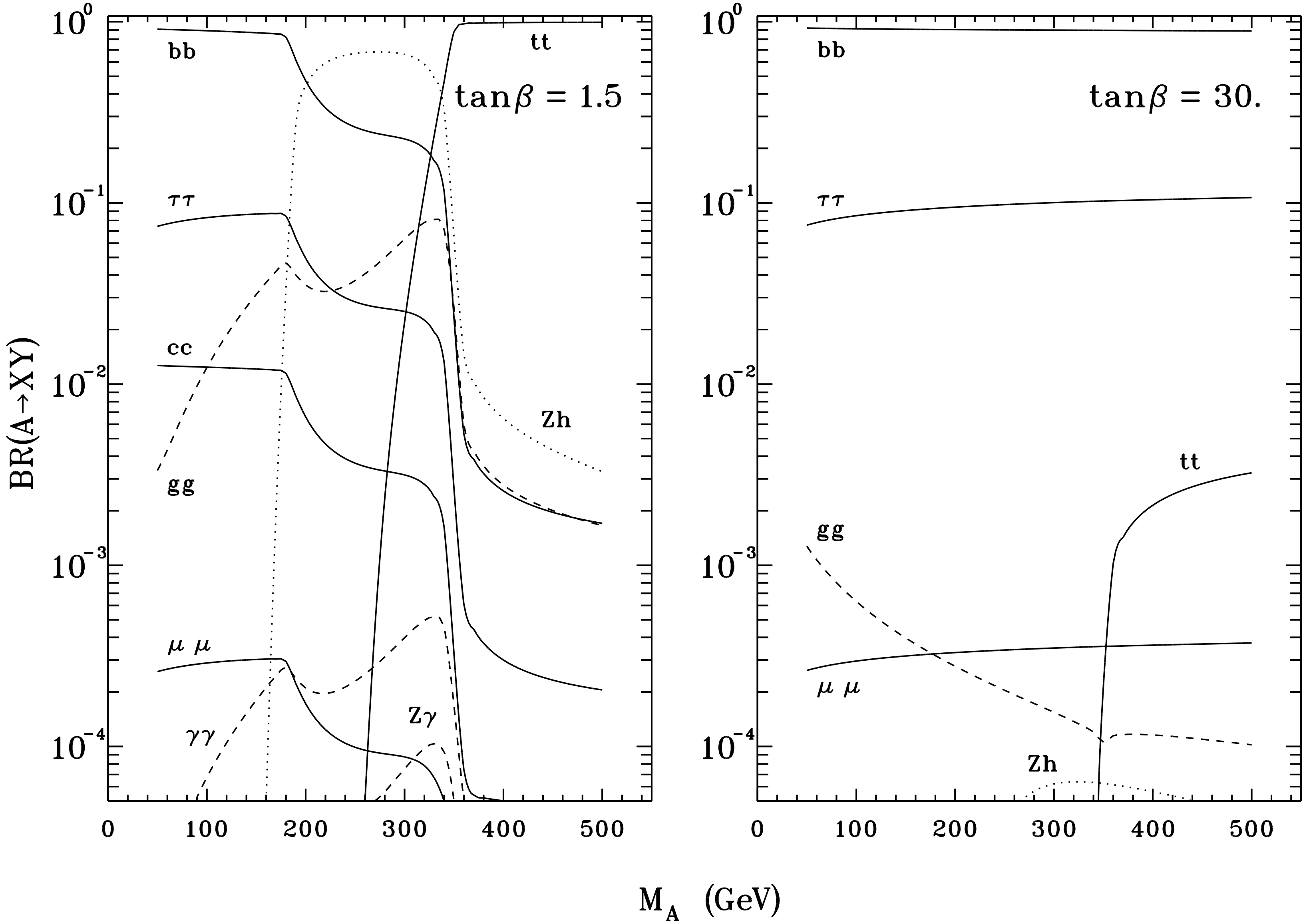

BR(A→XY)
tanβ = 1.5
bb
ττ
cc
gg
μ μ
γγ
tt
Zh
Zγ
tanβ = 30.
bb
ττ
gg
μ μ
tt
Zh
M_A (GeV)

Fig. 3

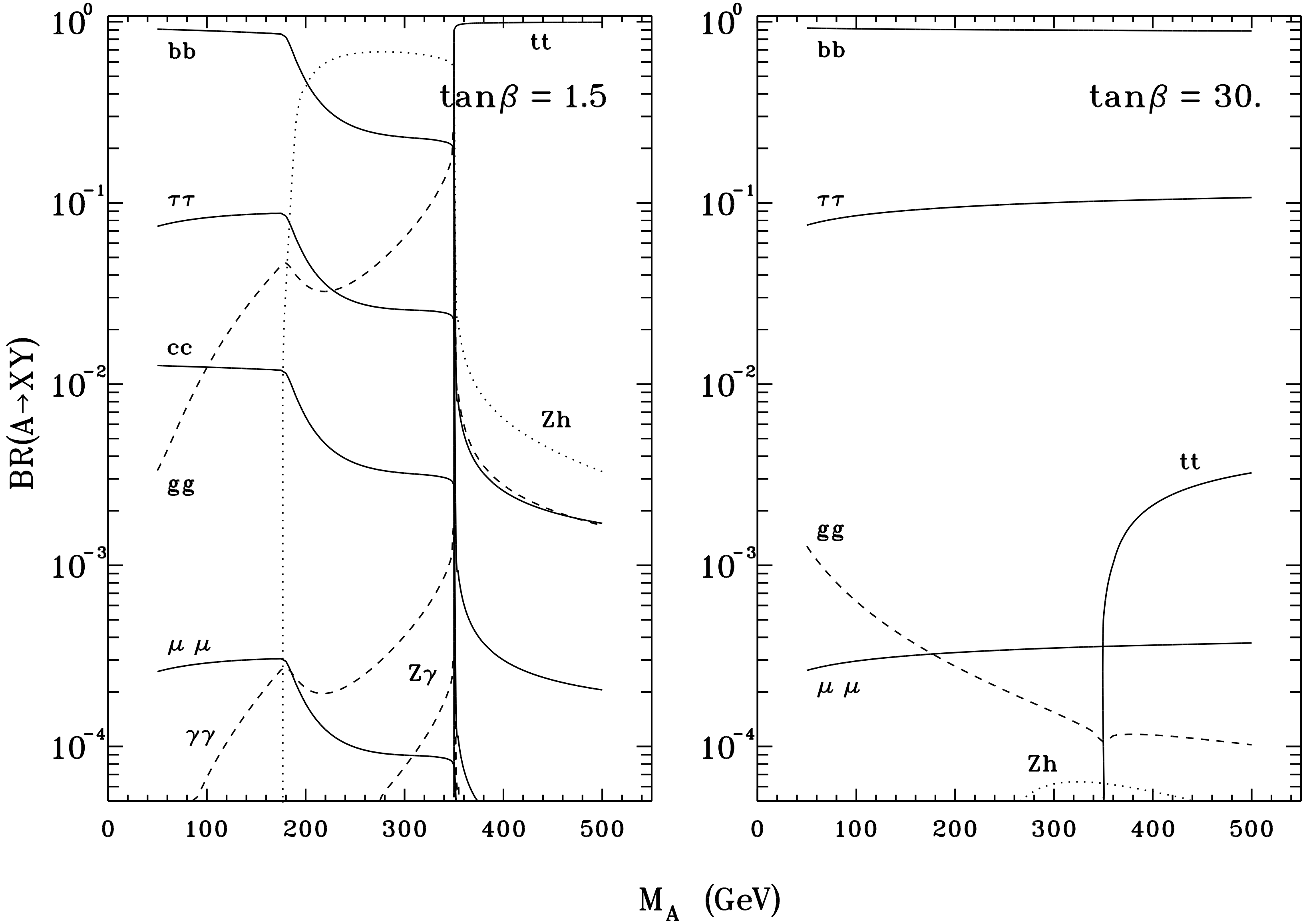
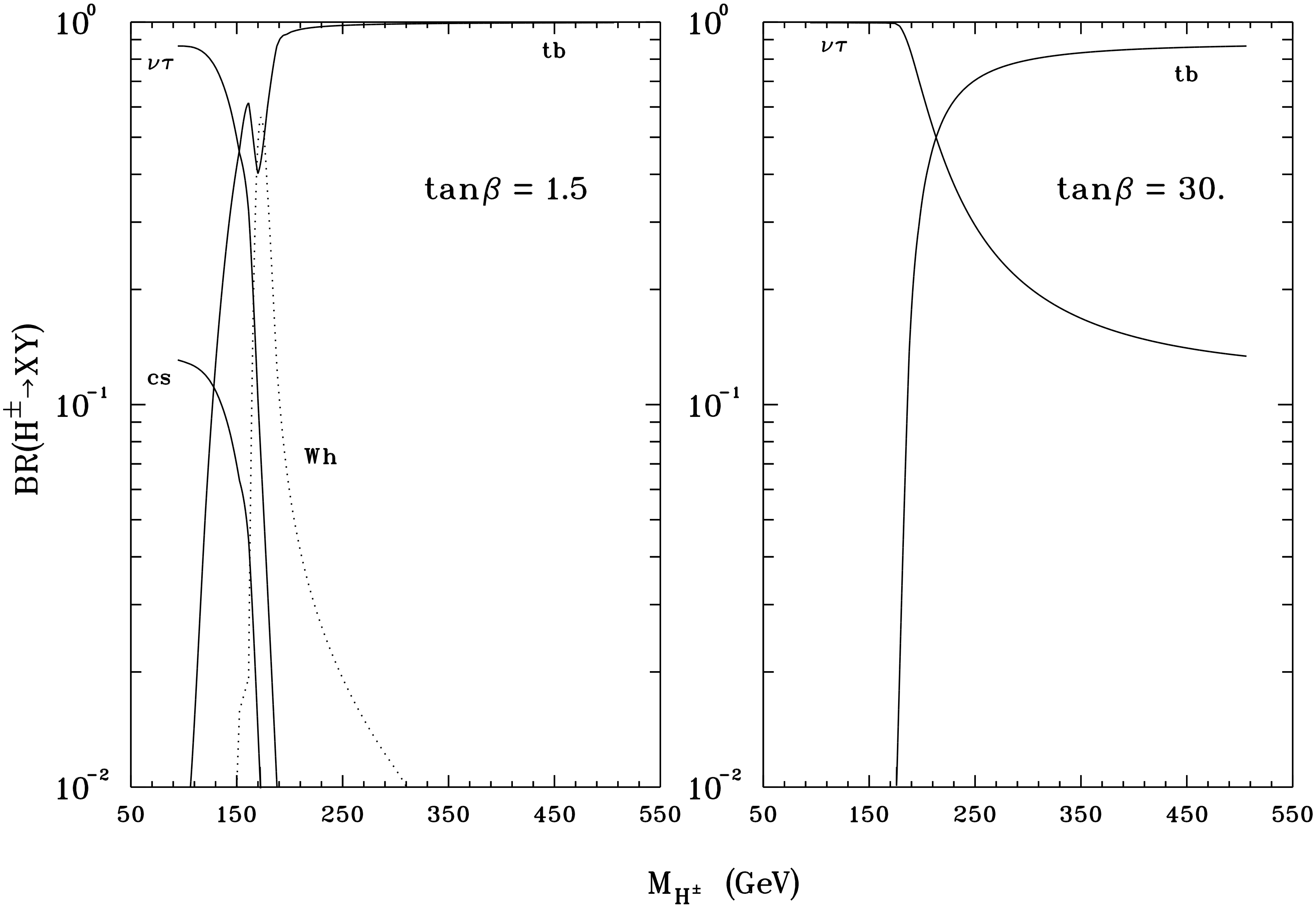


Fig. 4



BR(H$^\pm$→XY)
10^0
10^{-1}
10^{-2}
$\nu\tau$
cs
tb
Wh
tanβ = 1.5
tanβ = 30.
50
150
250
350
450
550
$M_{H^\pm}$ (GeV)

Fig. 5